Designing the US Incentive Auction

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The US government has begun a new kind of double auction to buy television broadcast rights from current TV broadcasters and sell mobile broadband licenses. Known as the "incentive auction" because it is intended to create an incentive for TV broadcasters to relinquish their licenses, this auction is likely to involve tens of billions of dollars of payments and must determine an allocation that satisfies millions of interference constraints. In this paper, we describe why the problem is so complex, outline the auction's final design adopted by the Federal Communication Commission, and describe how this design overcomes the novel challenges that the FCC has faced.

I. Background and Challenges

The incentive auction was mandated by an act of Congress in February 2012. In the years since the introduction of the iPhone, iPad and similar devices, there has been explosive growth in the demand for broadband services and the spectrum it uses. The US government responded with its National Broadband Plan, which aims to make more spectrum available for broadband services partly by clearing some bands currently in other uses. Some spectrum currently allocated to UHF television broadcasts is very well suited to broadband. Since 90% of US households currently have access to cable or satellite television, over-the-air broadcasting appears to be less important than in an earlier era. Also, the switch to digital TV signals has made it possible to transmit several standard-definition signals using a single TV channel.

The Roles of Government and Auction

Economists naturally ask: Why not just rely on private transactions to shift spectrum resources to their most valuable uses? Why is value added by a government intervention?

The answer is that spectrum reallocation has characteristics of a collective action problem. In order to use spectrum most effectively, the frequencies used for TV broadcasting need to be nearly the same across the whole country and the wireless broadband uses must be separated from TV frequencies and coordinated to avoid unacceptable interference between uplink and downlink uses.

Without government mandates, there could be an important broadcaster holdout problem. A single broadcaster on, say, channel 48 in some city could effectively block the use of spectrum on that channel and adjacent ones over a wide area. To resolve such problems, the law allows the FCC to require broadcasters that do not relinquish their licenses to move to a different broadcast channels, for example, from channel 48 to channel 26, provided the government makes "all reasonable effort" to ensure that the coverage area of the station is not substantially reduced by interference from other broadcasters. With the authority to retune broadcasters who do not relinquish their spectrum, FCC can make all broadcasters compete for the right to relinquish their licenses for compensation, regardless of the channels they are initially assigned (including channels that FCC does not intend to clear, but into which other broadcasters could be retuned). And some authority is needed to interpret and implement the words of the mandate of substantially preserving the post-repacking coverage of the remaining broadcasters.

To ensure that any transactions benefit the broadcasters, the law requires that participation in the incentive auction will be voluntary. Even broadcasters that do not sell their rights will benefit, because increased scarcity makes their remaining broadcast licenses more valuable. An auction tends to preserve high-value programming for consumers by encouraging only the lowest-value broadcasters to become sellers.

Special Challenges

Three technical characteristics make the incentive auction especially challenging compared to any previous two-sided auction.

First, the items being bought and sold – wireless broadband licenses and TV broadcast licenses – are not the same, so it is not sufficient to transfer a set of rights from seller to buyer. Compared to broadband licenses, television broadcast licenses involve different power restrictions, different coverage areas, different amounts of bandwidth, and different protections from interference. This makes both pricing and determining the quantities of each kind of license to buy and sell unusually challenging.

Second, the problem of moving television stations that do not sell to new channels is very complex. There are currently about 3,000 broadcast stations in the United States and Canada that may need to be repacked[1] and millions of pairwise constraints among them, mostly of the form: it is not allowed to assign station X to channel Y and station X' to channel Y', because that would cause unacceptable interference. Even if the bidders' values were known, the combinatorial problem of computing an optimal repacking would be computationally intractable.

But bidders' values are not known, and the auction needs to elicit information about them, and the practical impossibility of optimization implies that Vickrey prices, too, cannot be computed.[2] Paid-as-bid pricing would impose such intractable challenges for small bidders, whose participation is essential, that they might prefer to sit out the auction.

The FCC would also like to allow UHF broadcasters to continue broadcasting in the VHF band and VHF broadcasters to go off air, both in exchange for suitable compensation. Also, since the interference that remains after the auction depends on which stations are eventually cleared, some of the new mobile broadband licenses may be impaired to a degree that cannot be known before the auction. Uncertainty about the product is an enemy of a successful auction and needs to be minimized.

II. Three Part Plan and Desired Properties

The final auction design, based on the initial proposal of Milgrom et al. (2012) and finalized by Federal Communication Commission (2015), has three conceptual components: (i) a forward auction in which buyers will acquire broadcast licenses, (ii) a reverse auction in which sellers offer their broadcast licenses, and (iii) a coordination/ clearing rule to determine what quantities of licenses of each kind should be traded.

The design for the forward auction has a goal of reducing substantially the time to completion compared to the FCC's traditional simultaneous multiple round auction. It also aims to build the auction so that information elicited for one quantity target can be reused if the quantity target changes, so that multiple complete forward auctions will not be required.

[1] Canada has agreed to a joint repacking in order to also clear a swath of spectrum for mobile use and coordinate cross-border interference.

[2] Approximate optimization is possible but is not a suitable basis for strategy-proof pricing. This issue has been examined in the field known as algorithmic mechanism design. See Milgrom and Segal (2015) and the references therein.

The goal for the reverse auction is to acquire enough licenses to clear a pre-determined amount of spectrum at a reasonable cost, and to encourage participation by small broadcasters by making the auction nearly strategy-proof for them.

The clearing/coordination rule needs to ensure compatibility of the outcomes of the forward and reverse auctions and to account for the FCC's revenue objectives, which include raising funds to support a new public safety communications network.

III. The Economic Design

A. Forward Clock Auction

To achieve the speed-up required by the incentive auction, the forward auction is designed as a *clock auction* with an extra feature called *intra-round bidding* and with a separate *assignment round.* Clock auctions are conceptually similar to the FCC's familiar simultaneous multiple round auction, but with prices increasing on all licenses until demand falls to match the target supply.

To illustrate the difference, imagine a simultaneous multiple round auction for four identical licenses with five bidders and a limit of one license per bidder. At each round, there are four provisionally winning bidders and the fifth bidder raises the price of one license. It takes four rounds for the price of all four licenses to rise by one increment. A clock auction raises the price of all four licenses in every round so long as there is excess demand. In the incentive auction, it seems possible that this feature, alone, could reduce the auction time by about half.

Intra-round bidding is a feature that allows any bidder to indicate that it wishes to change its demand at prices between the proposed beginning and ending prices for a round, and prices generally stop rising when there is no excess demand for a product. In the preceding example, intra-round bidding allows the clock auction to use very large increments without overshooting the clearing price. Doubling the increment compared to the FCC's typical SMRA could cut the auction time in half again, so the combination of factors could shorten the auction duration by about 75%.

Because the licenses in a category in the clock auction are not actually identical, there needs to be a further procedure to determine which winner gets which licenses. In Europe, it has become common to use an assignment round to achieve that, and our design specified a similar procedure for the US.

For a multi-product auction, intra-round bidding is more subtle, but we omit the details here for reasons of length. Overall, a 75% reduction in time-to-completion appears to be a reasonable target for such auctions.

B. Reverse Clock Auction

In the reverse auction, each bidder will be offered tentative prices for several options that it could exercise, including to go off-air or, in exchange for compensation, to shift to broadcast in a lower band. The prices are initialized at high levels and are reduced over time, and during the auction each bidder indicates its currently preferred option. The initial clearing target is set to be consistent with the broadcaster's initial choices. To ensure that all transactions are voluntary, whenever a bidder's price for its preferred option is reduced, it must be given an opportunity to exit and so continue to broadcast in its original "home" band.

The clock auction will maintain a tentative feasible assignment of stations to bands and channels that avoids unacceptable interference. In every round, the auction must run a "feasibility checker" to determine whether there is still a feasible channel assignment when any given station is moved into any given band. If it is infeasible for a station to return to its home band, the auction cannot reduce the station's current price offer. Feasibility checking is an NP-hard problem similar to "graph coloring"; however, unlike the value-maximization problem, it can usually be solved in reasonable time. Since the constraints are known well in advance of the auction, modern computer science techniques can be used to develop a specialized algorithm that is expected to solve as many as 99% of potential feasibility checking problems within seconds (see Frechette et al. 2015). Also, feasibility checking for different stations can be done in parallel using hundreds of processors, as needed. When a checker has not found a feasible way to assign a station into a given band within the available time, the auction treats the move as infeasible.

Among the questions an economist might ask about this auction design are the following ones. (1) Can the auction achieve reasonably high efficiency and low cost of spectrum clearing? (2) How should clock price reductions be computed to advance those goals? (3) Does the auction avoid incentives for gaming? (4) How can it be integrated with the forward auction (i.e., how wills the amount of spectrum to clear be determined)? These questions have been examined both theoretically and in simulations.

Theoretical Analysis:

Equivalence Results — Milgrom and Segal (2015) (hereafter MS) offer a theoretical analysis which assumes that all bidders are single-station owners who know their station values and are "single-minded", that is, willing to bid only for a single option. This assumption is reasonable for commercial UHF broadcasters that view VHF bands as ill-suited for their operations and for non-profit broadcasters that are willing to move for compensation to a particular VHF band but that view going off-air as incompatible with their mission. A single-minded bidder may reasonably adopt a "cutoff strategy," in which it bids for its acceptable option until the compensation offered for that option reaches its minimum acceptable price, at which point it exits. MS finds that when bidders use such strategies, the class of clock auctions described above is equivalent to the class of sealed-bid *deferred-acceptance threshold auctions*, which use one of a class of deferred acceptance algorithms that prioritize bids for rejection by assigning a score to each bid at each round and rejecting the bid with the highest positive score until no such bids remain. The auction then pays each accepted bidder its "threshold price," which is the maximum price that bidder could bid that would have still been accepted, given the bids by the others.

Incentives. – What are the individual and group incentives for bidders in the reverse auction? MS show that for "single-minded" bidders, any clock auction with any information disclosure policy, or the equivalent sealed-bid DA heuristic auction with threshold prices, is *weakly group strategy-proof*: for every coalition of bidders and every possible strategy profile of bidders outside the coalition, there is no coalitional deviation from truthful bidding that strictly benefits all of its members.[3]

Advantages of Clock Auctions. In spite of the equivalence described above, clock auctions do offer some distinct advantages over the equivalent sealed-bid auctions:

1. In contrast to sealed-bid auctions with threshold pricing, the strategy-proofness of a clock auction is obvious even to bidders who do not understand the auction algorithm or trust the auctioneer's computations: as long as a bidder believes that his price offer is only going to

[3] The idea of the proof is simple: consider the first round of the clock auction in which a deviation occurs. If an agent deviates by exiting while his price offer exceeds his value, he receives a zero net payoff, while if he deviates by not existing when his price offer falls below his value, he will ultimately receive a nonnegative net payoff; in both cases, he does not strictly benefit from the deviation.

be reduced, it is clear that there is no gain to exiting while the price offer is above his value or to staying in the auction when your price offer is below your value. (A formal notion of "obvious strategy-proofness" that captures some of this intuition is offered by Li (2015).)

2. Unlike sealed-bid auctions, clock auctions do not require bidders to report or even know their exact values. For one thing, the winners in a clock auction will not have to evaluate their exact values: they only need to verify that their interested in selling at their final clock prices. In fact, MS show that for single-minded private-value bidders, clock auctions can be characterized by the property of "winners' unconditional privacy": they elicit minimal information about winners' values that is required to establish that they should be winning. Also, in the case of a common-value component (for example, this being the resale values of TV spectrum, about which different bidders could have different information), information feedback during the clock auction may help aggregate the common-value information among bidders, resulting in lower cost (cf. Milgrom and Weber, 1982).

3. Clock auctions can accommodate bidders who are interested in more than one option, by permitting them to switch between options as their prices are reduced. Such auctions for multi-minded bidders are generally no longer strategy-proof. For example, multi-station owners might profitably engage in "supply reduction," while bidders who are willing to switch between bidding to go off-air and bidding to move to a lower band might be able to influence its prices by choosing when to switch.[4] However, potential gains from such gaming would be small in a "large-market" setting in which there are many participants competing in each region. (FCC's current restrictions on station cross-ownership in each region help ensure such competitiveness.) Furthermore, a clock auction can provide some information feedback that could be useful for bidders to account for value complementarities or substitutabilities across their stations in their bidding strategies, but not detailed enough to facilitate effective price-manipulation strategies.

[4] There do exist clock auctions with package bidding multi-parameter bidders which sustain truthful bidding as an ex post Nash equilibrium, by implementing the Vickrey outcome and thereby eliminating the incentive for price manipulation (see, e.g., Ausubel and Milgrom, 2002). However, this only works when bidders are substitutes in the total value and when price adjustments are can be guided by exact optimization, and neither assumption holds for FCC's reverse-auction problem.

Price Reduction Algorithm for Efficiency and Cost Reduction – Given the equivalence result, a price reduction algorithm for single-minded bidders is equivalent to a scoring rule, and one could evaluate such scoring rules according to the efficiency of the resulting outcome and/or the sum of the threshold prices paid. If stations were always substitutes and optimization in reasonable time were possible, then as MS show, one can adapt results of Ausubel (2004) and Bikhchandani et al (2011) to show that a simple clock auction can implement the efficient allocation and determine Vickrey prices. In the actual problem, stations are not always substitutes, because it is sometimes possible to buy out two small stations instead of one large one, or to pay a UHF station to move to VHF and a VHF station to go off-air instead of paying a UHF station to go off-air. Also, given reasonable time limits, no algorithm can always find efficient allocations, or compute Vickrey prices. Nevertheless, as MS show, it may sometimes still be possible for some deferred acceptance threshold auction and its corresponding clock auction to be strategy-proof and guarantee near-efficiency.

One case in which a simple heuristic approach achieves great results is when all participating UHF stations interested only in off-air and not in going to VHF. The simplest problem arises when the interference graph is a union of regional "cliques" (a "clique" is a set of stations that all have same-channel interference constraints with each other) with no cross-clique constraints and no cross-channel constraints. Thus, assuming that there are k channels available in each region, the interference constraints allow us to assign any set of k stations, but no more than k, to broadcast in each region. In that case, a clock auction that offers the same descending price to all stations (i.e., no scoring) results in assigning the k most valuable stations in each region, which is efficient, and in paying each winner the value of the lowest assigned station in its region – the Vickrey price.

MS study an example in which the interference graph is more complicated than that. For example, some stations in Philadelphia stations may have interference constraints with New York stations, and others have constraints with Washington, D.C. stations. Nevertheless, MS show that if there are "not too many" cross-regional constraints then some simple clock auction would still be exactly or approximately efficient, assigning channels to the m highest-value stations in each region, with m less than k.

The actual interference constraints are even more complicated than that, and simulations suggest that it is possible to improve efficiency by reducing the scores of stations that have many interference constraints. In terms of the clock auction, this means offering a higher price to clear stations, which, if assigned, would cause more interference.

The problem of pricing VHF options and prices for VHF stations is substantially more complicated than the problem of pricing offers for UHF stations go off the air, as the problem of efficient repacking stations across bands may be less amenable to heuristic solutions. The price reduction algorithm designed for the auction, while not guaranteed to achieve a nearly-efficient repacking, extracts substantial additional value from VHF bands. The algorithm has the following properties:

A. The price offers to UHF stations to go off-air will be set proportionately to their "volumes" (determined by their population and interference), and reduced proportionately in each round the auction. This proportionate reduction will ensure a speedy auction (given the proposed price decrements, each stage of the reverse auction will finish in no more than 52 rounds).

B. The compensation offered to a station for going to a given band will never exceed the compensation offered to the same station for going to a lower band or for going off air. A station with a lower home band is never offered a higher compensation for a given option than an otherwise identical station with a higher home band. This reflects the consensus view that higher-frequency bands are more desirable for digital TV broadcasting.

C. For stations from different bands that have identical location, covered population, and interference, the auction will always equalize the total compensation for any combination of their moves that would clear a single UHF channel without changing the availability of channels in either VHF band.[5] This equalization is desirable for efficient utilization of VHF bands. In particular, it would link the price offers to a VHF station to those to an otherwise

[5] Such combinations include: (i) a UHF station going off the air, (ii) a UHF station going to upper VHF and an upper VHF station going off the air, (iii) a UHF station going to lower VHF and a lower VHF station going off the air, and (iv) a UHF station going to upper VHF, an upper VHF station going to lower VHF, and a lower VHF station going off the air.

identical UHF stations.

D. For any station, the difference between price offers for any two options could only decrease during the auction. This will ensure that stations without wealth effects (i.e., those whose objective is to maximize the sum of broadcast value and the compensation received in the auction) could only want to switch to a higher option as the auction progresses. The auction will also enforce an "activity rule" preventing stations from switching to a lower option than the one currently held, so as to eliminate the possibility of price manipulation by switching back and forth between options.[6]

E. The reduction in a station's price offer to a particular band is determined by the "vacancy" (the number of still-available channels in the tentative assignment) in the station's area in that band relative to the vacancy in the same area in other bands. The goal of this approach is to incentivize stations to fill different bands in each area at the same rate, in order to balance the possibility of the following two kinds of inefficiencies: (i) UHF stations "freezing" off-air before VHF is filled, leaving unused VHF channels to which UHF stations can no longer move, and (ii) VHF getting too congested while a lot of space is still left in UHF, which may induce low-value stations to move to VHF just because they fit well with the stations that are currently assigned there but are bound to eventually move to UHF.

The reverse auction is a component of a larger incentive auction. If clearing proves too costly relative to forward auction revenue, the clearing target would need to be reduced (as described in Section C below), which could entail substantial losses in overall efficiency. Therefore, cost-reduction measures may enhance overall efficiency even if they risk some efficiency distortions in deciding which stations to assign. One such cost-reduction measure that will be implemented will score bids based not only on the interference constraints the create but also on their covered population, which is an important determinant of the market value TV spectrum.

[6] However, this restriction will make it more difficult to bid for bidders with strong wealth effects, e.g., those who have a "revenue target" and so might wish to switch to a lower, better-compensated option as prices are reduced.

Paid-as-Bid Equivalence. – One might wonder whether a reverse clock auction, or the equivalent deferred acceptance threshold auction, could be criticized on the grounds that it "overpays" bidders relative to their willingness to accept. Of course, if a paid-as-bid auction is used instead, then reverse-auction bidders will bid above their values. In such a complicated setting, their exact bidding strategy and the resulting outcome is hard to predict. However, for a simple theoretical benchmark, suppose that bidders are single-minded, have full information about each other's values, and play a Nash equilibrium of the paid-as-bid auction. For this case, MS show that the paid-as-bid auction with a DA heuristic assignment rule has a full-information Nash equilibrium outcome that is the same as the truthful-bidding outcome of the threshold-price auction with the same heuristic assignment rule. In this sense, using threshold pricing "does not cost extra" given the heuristic assignment rule we are using.

The paid-as-bid auction typically has many Nash equilibria, but in certain cases, a unique outcome is selected by iterated deletions of weakly dominated strategies. MS shows this happens for deferred acceptance auctions that use "non-bossy" assignment rules – ones in which a losing bidder that switches to a different losing bid does not alter the set of winners.[7]

C. Coordination Clearing Rule

Having described the forward and reverse auction, there remains the task of coordinating them to buy and sell compatible sets of licenses. The FCC design coordinates these using a series of declining clearing targets, as follows.

The process would begin by setting reserve prices for broadcasters in the reverse auction. Broadcasters who register to bid would commit to sell at the reserve prices, which would also serve as the reverse auction starting prices. A rule would determine the number of broadcast channels that could be cleared across the country using the initial offers and a corresponding set of broadband licenses.

The forward and reverse auctions would follow, determining prices for both buyers and sellers. The auction would then check a stopping criterion: if net revenue (forward auction revenue minus reverse auction payments) is sufficiently high or if another FCC-determined criterion is satisfied,

[7] As an additional theoretical point, MS show that dominance-solvability of a paid-as-bid auctions with a non-bossy monotonic assignment rule is *equivalent* to the assignment rule being implementable with a DA heuristic (or a clock auction).

then the auction would end. The extra criterion might, for example, depend on the amount of spectrum cleared or on the average prices in the forward auction. Otherwise, the clearing target would be reduced, the products in the forward auction would be adjusted, and prices would continue to descend in the reverse auction and ascend in the forward auction.

Another novel feature of the forward auction is its use of "extended rounds," which is triggered if the auction revenue is still too low when demand has fallen to be equal to target supply. In that case, provided that the revenues are within 25% of the target, prices in the forward auction would continue to rise, in order to give bidders in the forward auction an opportunity to raise their bids to meet the prices set in the reverse auction. Details are not reported here.

A final novel feature of the design is the "conditional reserve." The FCC faced a novel challenge between promoting competition among wireless carriers and raising sufficient revenues to satisfy the stopping criterion. On one hand, the FCC wanted to ensure that the low-band spectrum, including that sold in this auction, was not all held by just one or two companies, who might acquire too much market power in that way. On the other hand, if the FCC were to limit competition in the auction by setting aside spectrum for bidders without existing holdings of low-band spectrum, the forward auction revenues might be reduced, and it might fail to clear a large number of TV channels. The solution the FCC adopted is the conditional reserve, according to which spectrum is set aside only when the net revenue condition from the stopping criterion is met. Moreover, the number of licenses to be set aside would be the smaller of a pre-determined number or the number of licenses being demanded by qualified bidders at the time the criterion is satisfied. In that way, the set aside would not threaten clearing, but would be effective at encouraging competition once the clearing condition had been met.

IV. Conclusion

The FCC incentive auction poses unusual challenges but also promises a rare opportunity to contribute to the wireless broadband infrastructure of the United States. We are excited to be part of the team that designed this new auction.

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